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$\mathcal{A}(1^{L_i}, 2, 3, \ldots, n)$ L_i: little groups.

① Assume asymptotic states → [⊗ Single particle states] →

not necessarily true (monopoles)

L_i → Massive $p^\mu = (E, \underbrace{0, 0, \cdots}_{SO(D-1)})$ $D=4$ $SO(3) \sim SU(2)$ ✓

→ Massless $p^\mu = (E, \underbrace{0, 0, \cdots,}_{SO(D-2)} E)$ $D=4$ $SO(2) \sim U(1)$ ✓

$\otimes \, | P_i^\mu, \underbrace{\{I_1 I_2 \cdots I_{2S}\}}_{\text{spin}-S} \rangle$ Massive $|P_i^\mu, \underset{\text{helicity}}{h}\rangle$

When one has electric and magnetic charged objects (pairwised helicity)

1 2

pairwise little group (P_1, P_2) unchanged

leaves

C.O.M frame $P_1 = (E_1, 0, 0, \ldots, P)$

$P_2 = (E_2, 0, 0, \cdots, -P)$

$SO(D-2)$ 4D $= SO(2) \sim U(1)$

$\mathcal{A}(P_1^\mu, P_2^\mu, \cdots P_n^\mu ; S_1, S_2, h_3, h_4, \cdots ; \cancel{\{h_{12}\}, \cdots}) = f(\{P_i^\mu\}, \epsilon, \ldots)$

complete set of quantum numbers.

Text book : external line factors $\epsilon_\mu^\pm$ $\epsilon^\pm \cdot P = 0$

$v^{\pm}_{\alpha}$ $\not{p} v^\pm = 0$

$\epsilon_{\mu\nu}^\pm$ $P^\mu \epsilon_{\mu\nu} = 0$

Here : not introduce redundancies

$$P_i^{\mu} \longrightarrow P_i^{\alpha\dot\alpha} = P_i^{\mu}(\sigma_\mu)^{\alpha\dot\alpha} = \begin{bmatrix} P_i^0+P_i^3 & P_i^1+iP_i^2 \\ P_i^1-iP_i^2 & P_i^0-P_i^3 \end{bmatrix}$$

$$\det[P_i^{\alpha\dot\alpha}] = \underline{(P_i^0)^2-(P_i^1)^2-(P_i^2)^2-(P_i^3)^2}.$$

Massless : $\det[P_i^{\alpha\dot\alpha}]=0 \rightarrow \underline{P_i}^{\alpha\dot\alpha}$ rank 1 $= \lambda_i^{\alpha}\tilde\lambda_i^{\dot\alpha}$

$= \vec\lambda_i(\vec\lambda_i)^{\dagger}$

$$\lambda_i^{\alpha} \rightarrow e^{-i\frac{\theta}{2}}\lambda_i^{\alpha}$$
$$\tilde\lambda_i^{\dot\alpha} \rightarrow e^{i\frac{\theta}{2}}\tilde\lambda_i^{\dot\alpha}$$

$P_i^{\alpha\dot\alpha} \rightarrow P_i^{\alpha\dot\alpha}$

$\lambda_i, \tilde\lambda_i$ carries little group weight.

$\lambda_i : -\frac{1}{2}$

$\tilde\lambda_i : +\frac{1}{2}$

Massive : $\det[P_i^{\alpha\dot\alpha}] = m_i^2. \rightarrow \underline{P_i}^{\alpha\dot\alpha}$ $\underline{\text{rank 2}}$ $= \lambda_i^{\alpha}\tilde\lambda_i^{\dot\alpha} + u_i^{\alpha}\tilde u_i^{\dot\alpha}$

$= \boxed{\lambda_i^{I\alpha}\tilde\lambda_{iI}^{\dot\alpha}}$ $\quad I=1,2$

$SU_L(2)$ indices $\quad \alpha=1,2$

$\dot\alpha=1,2$

$$\lambda^{I} = \epsilon^{IJ}\lambda_J$$

$$\lambda_i^{I\alpha} \rightarrow (\underline{L})^{I}{}_{J}\,\lambda_i^{J\alpha}$$
$$\tilde\lambda_{iI}^{\dot\alpha} \rightarrow (\underline{L})_{I}{}^{J}\,\tilde\lambda_J^{\dot\alpha}$$

$$A\left[\{\underline{\lambda_i, \tilde\lambda_i}\}_{\text{massless}}, \{\underline{\lambda_j^I, \tilde\lambda_{jI}}\}_{\text{massive}}\right]$$

$\rightarrow$ Lorentz scalar

$\alpha, \dot\alpha$ $SL(2,C)$ $\longrightarrow$ Invariants contract α,β $\epsilon^{\alpha\beta}$

$\dot\alpha$ $\dot\beta$ $\epsilon^{\dot\alpha\dot\beta}$

Lorentz invariant building blocks are.

$$\underline{\lambda_i^\alpha} \underline{\lambda_j^\beta} \epsilon_{\alpha\beta} = \lambda_i^\alpha \lambda_{j\alpha} = \underline{\langle i j \rangle}$$

$$\tilde\lambda_i^{\dot\alpha} \tilde\lambda_j^{\dot\beta} \underline{\epsilon_{\dot\alpha\dot\beta}} = [i j] \rightarrow [i\, i] = 0$$

$$\lambda_i^\alpha \lambda_j^{\beta I} \epsilon_{\alpha\beta} = \langle i j^I \rangle$$

$$\tilde\lambda_i^{\dot\alpha} \tilde\lambda_{jJ}^{\dot\beta} \epsilon_{\dot\alpha\dot\beta} = [i\, j_J]$$

$$\lambda_i^{\alpha I} \lambda_j^{\beta J} \epsilon_{\alpha\beta} = \langle i^I j^J \rangle \rightarrow \langle i^I i^J \rangle = M_i \epsilon^{IJ}$$

$$- \quad - \quad \backslash \qquad [i_I\, j_J] \rightarrow [i_I\, i_J] = M_i \epsilon_{IJ}$$

$$S_{ij} = \underline{2 P_i \cdot P_j} = \underline{\langle ij \rangle} \underline{[j i]} \quad (\text{massless})$$

Massless

$$A_n (\langle ij \rangle, [ij])$$

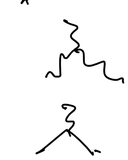
$\rightarrow$ All massless 3-pt amplitudes (interactions) are unique

$$A_3 \left[\begin{matrix} \langle 12 \rangle, \langle 23 \rangle, \langle 31 \rangle \\ [12], [23], [31] \end{matrix} \right] = A_3 (\underline{P_1, P_2, P_3, h_1, h_2, h_3})$$

quantum numbers

$+1 \ +1 \ -1$

$+1 \ +\frac{1}{2} \ -\frac{1}{2}$



① Momentum conservation $\quad P_1 + P_2 + P_3 = 0 \rightarrow (P_1 + P_2)^2 = 0$

$$P_1 \cdot P_2 = 0 \Rightarrow \langle 12 \rangle [21] = 0$$

$\langle 12 \rangle = 0$ $\quad \epsilon_{\alpha\beta}\lambda_1^\alpha \lambda_2^\beta = \det[\vec{\lambda}_1, \vec{\lambda}_2] = 0 \rightarrow \vec{\lambda}_1 // \vec{\lambda}_2$

$\langle 23 \rangle = 0$

$\langle 31 \rangle = 0$

$\lambda_1^\alpha \propto \lambda_2^\alpha$

$p_1^{\alpha\dot\alpha} + p_2^{\alpha\dot\alpha} = -p_3^{\alpha\dot\alpha}$

$\lambda_3 \propto \lambda_1^\alpha$

$\rightarrow \lambda_1^\alpha \tilde\lambda_1^{\dot\alpha} + \lambda_2^\alpha \tilde\lambda_2^{\dot\alpha} = -\boxed{\lambda_3^\alpha}\tilde\lambda_3^{\dot\alpha}$

$\lambda_1^\alpha \tilde\lambda_1^{\dot\alpha} + b\lambda_1^\alpha \tilde\lambda_2^{\dot\alpha} = \boxed{\lambda_1^\alpha}(\tilde\lambda_1^{\dot\alpha} + b\tilde\lambda_2^{\dot\alpha})$

either all $\langle ij \rangle = 0$

or all $[ij] = 0$

② $A_3(\cdots, h_1, h_2, h_3)\Big|_{\text{little group leg 1}} \Rightarrow e^{ih_1\theta} A_3$

$2 \Rightarrow e^{ih_2\theta} A_3$

$\boxed{\text{Assume } [ij] = 0}$

$\langle 12 \rangle \quad \langle 23 \rangle$

$\langle 13 \rangle$

$\langle 12 \rangle^a \langle 23 \rangle^b \langle 31 \rangle^c$

$-\frac{a}{2} - \frac{c}{2} = h_1$

$-\frac{a}{2} - \frac{b}{2} = h_2$

$-\frac{b}{2} - \frac{c}{2} = h_3$

Exp: $h_1 = -1 \quad h_2 = -1, \quad h_3 = +1$

$-2 \qquad -2 \qquad +2$

$\frac{\langle 13 \rangle^6}{\langle 23 \rangle^2 \langle 13 \rangle^2}$ ✓ $= \left[\frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 13 \rangle}\right]^2 \quad \frac{\theta^3}{\theta^2}$ or $\cancel{\frac{[23][13]}{[12]^3}}$ ✓ $\leftarrow \frac{\theta^2}{\theta^3}$

Grav $= [YM]^2 \quad h_1 = -1 \quad h_2 = -1 \quad h_3 = -1$

✓ $\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle$ or $\cancel{\frac{1}{[12][23][31]}}$ ✓

Assume $\langle ij \rangle = 0$

$F^2 \to \underline{A \partial A A}$

F^3

$D^2 F^3$

Four particle amplitude.

$$A_4 (1^{h_1} 2^{h_2} 3^{h_3} 4^{h_4}) \leftarrow \text{a function of } \langle ij \rangle, [ij]$$

Locality $\to$ the singularity must be assciated with.

$\frac{1}{(P^2)^*}$ $\quad$ $\partial = \partial\partial\partial\partial$

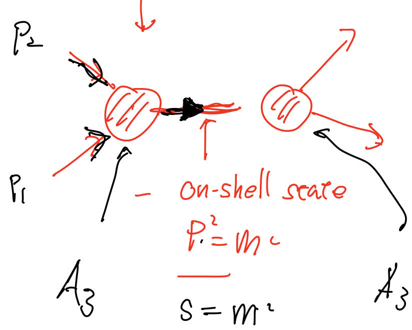
single poles, $s, t, u = m^2$ or branch cuts in s, t, u,

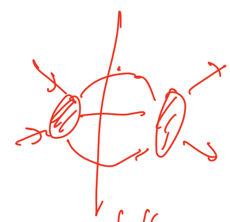
$S = (P_1 + P_2)^2$

$t = (P_1 + P_4)^2$

$u = (P_1 + P_3)^2$

$$s + t + u = \sum_{i=1}^{4} m_i^2 = 0$$





$$\text{Res}\,[A_4]_{s=m^2} = [A_3] \otimes [A_3]$$

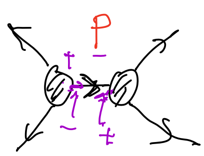
$$A_4 = \frac{[A_3] \otimes [A_3]}{s - m^2} + \text{Regular}_i\,(s = m^2)$$

$m^2 = 0$

Exp: $A_4(1^{-h}\, 2^{+h}\, 3^{-h}\, 4^{+h})$ self-interacting spin-h.

$P = -P_1 - P_2$
$= P_3 + P_4$

$s = 0$ $\qquad$ $\text{Res}\,[A_4]_{s=0} = A_3(1^{-h}, 2^{+h}, P^{\pm h})\, A_3(-P^{\mp h}, 3^{-h}, 4^{+h})$



consider $A_3(1^{-h}\, 2^{+h}\, P^{+h})\, A_3(-P^{-h}\, 3^{-h}\, 4^{+h})$

$\langle ij\rangle = \lambda_i^{\alpha}\lambda_{j\alpha}$

$\langle -i\, j\rangle = i\,\lambda_i^{\alpha}\lambda_{j\alpha}$

$-P_i = \lambda_{-i}\tilde{\lambda}_{-i} = i\lambda_i\, i\tilde{\lambda}_i = -\lambda_i\tilde{\lambda}_i = -P_i$

$$= \boxed{\frac{[2P]^{3h}}{[12]^h[1P]^h}\;\frac{\langle -P3\rangle^{3h}}{\langle 34\rangle^h\langle -P4\rangle^h}}$$

$$= \left(\frac{[2P]\langle P3\rangle^{3}}{[12][1P]\langle 34\rangle\langle P4\rangle}\right)^h$$

$$[2P]\langle P3\rangle = \tilde{\lambda}_{2\dot\alpha}\boxed{\tilde{\lambda}_P^{\dot\alpha}\,\lambda_P^{\alpha}}\lambda_{3\alpha}$$

$[2|P|3\rangle$

$$P^{\dot\alpha\alpha} = -P_1^{\alpha\dot\alpha} - P_2^{\alpha\dot\alpha} = P_3^{\alpha\dot\alpha} + P_4^{\alpha\dot\alpha}$$

$$= [2|-P_1-P_2|3\rangle$$

$$= [2|P_1|3\rangle + [2|P_2|3\rangle$$

$$= [21]\langle 13\rangle + \underbrace{[22]}_{0}\langle 23\rangle = [21]\langle 13\rangle$$

$$\left(\frac{[2P]\langle P3\rangle^{3}}{[12][1P]\langle 34\rangle\langle P4\rangle}\right) = \frac{[21]^{\cancel{3}}\langle 13\rangle^3}{\cancel{[12]}\langle 34\rangle\,\cancel{[12]}\langle 24\rangle}$$

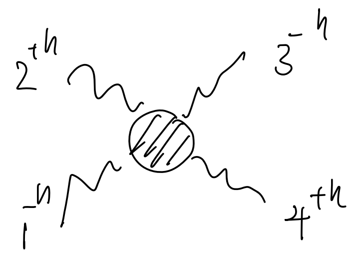
$\tilde{\lambda}_1\lambda_1 = P_1 = -P_2 - P_3 - P_4$

$$= \frac{[21]\langle 13\rangle\,\langle 13\rangle^2}{\langle 34\rangle\langle 24\rangle} = \frac{-[24]\cancel{\langle 43\rangle}\langle 13\rangle^2}{\cancel{\langle 34\rangle}\langle 24\rangle}$$

$$= -\frac{[24]^2\langle 13\rangle^2}{\langle 24\rangle[42]} = \boxed{\frac{[24]^2\langle 13\rangle^2}{u} \sim -t}$$

$s=0 \qquad s+t+u=0 \to t=-u$

A_4



$$\mathrm{Res}\,[A_4]_{s=0} = \left(\frac{[24]^2\langle 13\rangle^2}{t}\right)^h C$$

$$\mathrm{Res}\,[A_4]_{t=0} = \left(\frac{[24]^2\langle 13\rangle^2}{s}\right)^h C'$$

$$\mathrm{Res}\,[A_4]_{u=0} = \left(\frac{[24]^2\langle 13\rangle^2}{t}\right)^h C''$$

$h=0$ $\quad A_4 = c \times \left[\frac{1}{s} + \frac{1}{t} + \frac{1}{u}\right]$ self-interacting massless scalar.

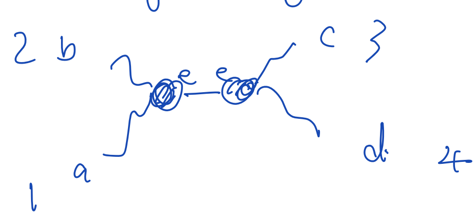
$h=1$: $\quad A_3(1^{+1a} 2^{+1b} 3^{-1c}) = \frac{[12]^3}{[13][23]} f^{abc}$ ← anti-symmetric in $1 \leftrightarrow 2$

$1 \leftrightarrow 2$
$a \leftrightarrow b$

$[12] = -[21]$

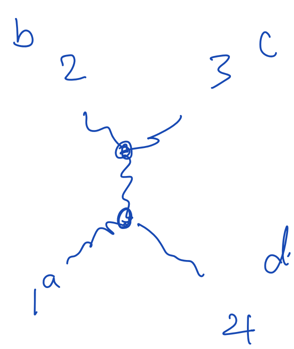
$\tilde\lambda_1^{\dot\alpha}\, \tilde\lambda_2^{\dot\beta}\, \epsilon_{\dot\alpha\dot\beta}$

f^{abc} totaly anti-symmetric.



$\mathrm{Res}\,[A_4]_{s=0}$

$$= \boxed{\frac{\langle 13\rangle^2[24]^2}{t} f^{abe} f^{cde}}$$

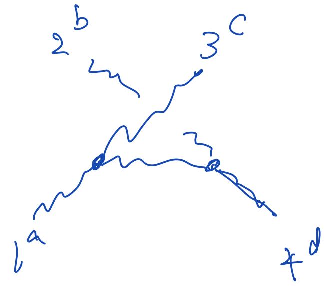


$\mathrm{Res}\,[A_4]_{t=0}$

$$= \frac{\langle 13\rangle^2[24]^2}{s} f^{bce} f^{dae}$$

$$\mathrm{Res}\left[A_4\right]_{u=0}$$

$$= \frac{\langle 13\rangle^2 [24]^2 f^{ace} f^{bde}}{t}$$



$$A_4 = \langle 13\rangle^2 [24]^2 \times \left(\frac{A^{abcd}}{s\,t} + \frac{B^{abcd}}{t\,u} + \frac{C^{abcd}}{s\,u} \right) + \text{Regular terms } f(s,t,u)$$

$$\frac{C^{abcd}}{s\,u} \to -\frac{C^{abcd}}{t}$$

$s=0$: $$A^{abcd} - C^{abcd} = f^{abe} f^{cde}$$

$t=0$: $$B^{abcd} - A^{abcd} = f^{bce} f^{dae}$$

$u=0$: $$C^{abcd} - B^{abcd} = f^{ace} f^{dbe}$$

$$f^{abe} f^{cde} + f^{bce} f^{ade} + f^{cae} f^{bde} = 0$$

Jacobi-identity!

$$\mathrm{tr}\left([T^a, T^b]\, T^c\right) = f^{abc}$$

We find consistent factorization of A_4 → self interacting spin-1 is associated with Lie-2 Algebra

$h=2$ 3-point amp. $A_3(1^{-2}\, 2^{-2}\, 3^{+2}) = \dfrac{\langle 12\rangle^6}{\langle 23\rangle^2\langle 13\rangle^2}$

symmetric under $1 \Leftrightarrow 2$ exchange

self interacting single spin-2 is consistent at 3 pt.

$$\mathrm{Res}\,[A_4]_{s=0} = \frac{\langle 13\rangle^4 [24]^4}{t^2} \longleftrightarrow \frac{\langle 13\rangle^4 [24]^4}{-tu}$$

double pole

$s+t+u=0 \rightarrow s=0 \quad t=-u$

$$A_4 = \boxed{\frac{\langle 13\rangle^4 [24]^4}{s\,t\,u}}$$

E.H.

$h>2$

$$\mathrm{Res}\,[A_4] = \frac{\langle 13\rangle^h [24]^h}{t^h}$$

$h = 2$
$\quad\ \ 3$

There is no solution to factorization constraints that is comprised of only single poles (unitarity)!

$h=0$

$$A_4 = \left(\frac{1}{s} + \frac{1}{t} + \frac{1}{u}\right) + f(s,t,u)$$

$$\boxed{\sum_{k,q} g_{k,q}\, s^{k-q}\, t^{q}} \leftarrow \begin{matrix}\phi^4 \\ D^2\phi^4 \\ D^4\phi^4\end{matrix}$$

$h=1$

$$A_4(1^{-1}\, 2^{+1}\, 3^{-1}\, 4^{+1}) = \langle 13\rangle^2 [24]^2 \times \left(\frac{A^{abcd}}{st} + + \right)$$

$$\langle 13\rangle^2 [24]^2 \left[\underbrace{\sum_{k<q}}_{} g_{k,q}\, s^{k-q}\, t^{q} \right]$$

E^4

$D^2F^4 \quad D^6F^4 \cdots$

$h=2$

$$\langle 13\rangle^4 [24]^4 \times \left[\frac{1}{stu} + \sum_{k,q} g_{k,q}\, s^{k-q}\, t^{q} \right]$$

$\downarrow$

$D^{2n}R^4$